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Optimization of laminates with free edges under bounded uncertainty subject to extension, bending and twisting

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Abstract

The layup of the maximum strength of laminated composites with free edges under extension, bending, and twisting loads was optimized by genetic algorithm (GA). Interlaminar stresses, as well as in-plane stresses, were considered in estimating the strength of laminates. To calculate the interlaminar stresses of composite laminates with free edges, iterative stress-based method was applied. To consider the bounded uncertainty of material properties, convex modeling was used. Because interlaminar strength can change rapidly with respect to the scattered deviation of material properties values, a linear convex model is not suitable in considering bounded uncertainty. Thus, in the present study, two-point exponential approximation was used to build a convex set. In the formulation of a GA, a repair strategy was adopted to satisfy given constraints. In addition, a multiple elitism was implemented to efficiently and reliably search the optimum and near optimal designs as many as possible. Because uncertainties are always encountered in composite materials, they need to be taken into account in lightweight design of laminated composite structures. The combination of genetic algorithms and convex modeling is practical in accounting for the uncertainties and optimizing layup.

Keywords: Interlaminar stress; Genetic algorithm; Repair strategy; Multiple elitism; Convex modeling; Two-point exponential approximation (TPEA)

1. Introduction

Laminates with free edges experience the failure initiations by the critical interlaminar stresses near the free edges. Thus, if the presence of free edges cannot be avoided, layups should be arranged such that the interlaminar stresses are suppressed near the free edges of laminates. The interlaminar free-edge strength analysis plays a key role in solving the optimization problem of a layup design of laminates. Although the current finite element method can predict the interlaminar stresses, it still requires much computation time and quite often it cannot predict accurately the interlaminar stresses near the interface between layers. In

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addition, the repeated efforts to prepare different meshes for analyzing the behaviors for various layup configurations make design optimization not only impossible but much harder. Thus, a simple and efficient analytical method (Cho and Yoon, 1999; Cho and Kim, 2000) should be designed to analyze the interlaminar stresses near the free edges. Kim et al. (2000) demonstrated the efficiency and reliability of the method in predicting the free-edge interlaminar stresses and strength under thermo-mechanical loadings. The iterative extended Kantorovich method was used in the present study to analyze the strength of free edges.

In laminated composite structures, the layups of laminates can be arranged to be lightweight and/or to have high performance. In most structural designs using composite laminates, laminates are restricted to some discrete sets of ply orientation angles such as 0°, ±30°, ±45°, ±60° and 90°. This practical manufacturing point of view requires the discretized optimization methodology for the layup design problem. Genetic algorithm (GA) or simulated annealing method is considered an appropriate method for discretized optimization problems.

Genetic algorithm is a powerful methodology for problems with integer variables and problems in which the gradient of the objective function is difficult to obtain (Goldberg, 1989). Recently, much research in design optimization of composite structures employing a genetic algorithm has been reported. The application of genetic algorithm for composite structures was initially reported by Hajela (1990). Le Riche and Haftka (1993) proposed a genetic algorithm to optimize the stacking sequence of a composite laminate for maximum buckling load. For the same problem, Liu et al. (2000) provided permutation genetic algorithms. Le Riche and Haftka (1995) used a genetic algorithm for the minimum thickness design of composite laminated plates. They proposed improved selection, mutation, and permutation operators to reduce the average price of a genetic search. Nagendra et al. (1996) proposed an improved genetic algorithm to design stiffened composite panels. A recessive-gene-like repair strategy was introduced by Todoroki and Haftka (1998) to handle given constraints efficiently. Recently, Soremekun et al. (2001) applied the generalized elitist selection (GES) to the problems with many global optima and to those with many designs that show performance very close to optimal design.

In the present study, genetic algorithm with the repair strategy is adopted. The balanced symmetric layup constraints and limitation of four contiguous layers are implemented by the repair strategy. To find multiple global optima, we selected the first multiple elitist selection (ME1) among GES.

Due to the uncertainty of the manufacturing process on the laminated composites, the stiffnesses, angle orientations and ply thicknesses of laminates are not determined uniformly. The scattering of these parameters depends on the quality of cured laminates. These uncertainties should be included in the design process. However, the information on the overall probability density function for each of the parameters is difficult to obtain. Thus, we postulate that the scattering bounds with respect to the nominal values of the parameters such as material properties are known priori. Then, by constructing a convex set from the scattering bounds and sensitivities of the functional, a modified functional with the effect of uncertainties can be obtained (Ben-Haim and Elishakoff, 1990; Elishakoff and Colombi, 1993; Elishakoff et al., 1994; Kim and Sin, 2000).

A linear convex modeling should not be applied to estimate the interlaminar free-edge strength, which changes rapidly with respect to the deviation in the material property values. Thus, in the present study, convex modeling for interlaminar strength under bending and twisting is constructed by using information from the interlaminar failure surface which is approximated by two-point exponential approximation (TPEA) (Fadel et al., 1990). Previously, the optimization considering uncertainties of material properties under extension was presented by Cho and Rhee (in press). In the present study, extension, bending, and twisting loads are considered to handle various loading conditions.

The present study consists of the following: First, the extended Kantorovich method for free-edge stress/strength analysis is outlined for stretching, bending, and twisting. Second, the formulation of a modified optimized functional subject to a convex set of constraints is derived with TPEA. Third, genetic algorithm with a repair strategy and a multiple elitism is outlined. Finally, numerical examples and discussions are provided.

2. Strength analysis considering free-edge strength

2.1. Extended Kantorovich method

The geometry of composite laminates with free edges under extension, bending, and twisting is given in Fig. 1. The laminate consists of orthotropic materials. The thickness of each ply is all the same, and symmetric layups are considered. The linear elastic constitutive equations are assumed in each ply, and they are expressed in the following form,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ S_{12} & S_{22} & S_{23} & 0 & 0 & S_{26} \\ S_{13} & S_{23} & S_{33} & 0 & 0 & S_{36} \\ 0 & 0 & 0 & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{45} & S_{55} & 0 \\ S_{16} & S_{26} & S_{36} & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ \alpha_6 \end{Bmatrix} \Delta T. \tag{1}$$

For the given geometric configuration of laminates, the boundary conditions at the free edge and at the surfaces of top and bottom faces are given in Eq. (2).

$$\begin{aligned} &\sigma_2 = \sigma_4 = \sigma_6 = 0 \quad \text{at } y = 0, b, \\ &\sigma_3 = \sigma_4 = \sigma_5 = 0 \quad \text{at } z = \pm h/2. \end{aligned} \tag{2}$$

Generalized plane strain states are assumed and the stress fields are independent of x-axis. The coordinates are nondimensionalized as follows,

$$\eta = z/h, \quad \xi = y/h.$$

Lekhnitskii stress functions are employed to satisfy pointwise equilibrium equations automatically. These stress functions can be divided into the in-plane and out-of-plane functions. The functions $f_i(\xi)$ and

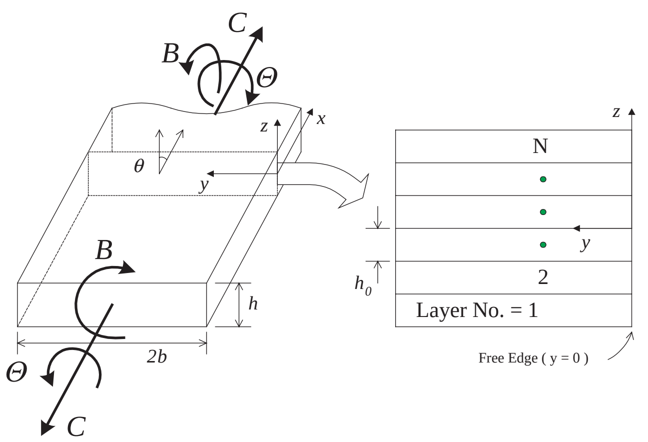

Fig. 1. Geometry of composite laminates with free edges.

$p_i(\xi)$ are in-plane functions, and $g_i(\eta)$ and $h_i(\eta)$ are out-of-plane functions. The individual stress components are obtained from Lekhnitskii stress functions and the relationships are given as,

$$\begin{aligned}
&\frac{\partial^2 F}{\partial \eta^2} = \sigma_2, \quad \frac{\partial^2 F}{\partial \xi^2} = \sigma_3, \quad \frac{\partial^2 F}{\partial \eta \partial \xi} = -\sigma_4, \\
&\frac{\partial \psi}{\partial \xi} = -\sigma_5, \quad \frac{\partial \psi}{\partial \eta} = \sigma_6,
\end{aligned} \tag{3}$$

where,

$$F = \sum_{i=1}^{n} f_i(\xi) g_i(\eta), \quad \psi = \sum_{i=1}^{n} p_i(\xi) g_i^{\mathrm{I}}(\eta) \tag{4}$$

for extension and bending loads. The superscript I in Eq. (4) denotes differentiation with respect to η. Or,

$$F = \sum_{i=1}^{n} f_i(\xi) g_i(\eta), \quad \psi = \sum_{i=1}^{n} p_i(\xi) h_i(\eta) \tag{5}$$

for twisting load. Since assumed stress functions $g_i^{\mathrm{I}}(\eta)$ in Eq. (4) cannot provide nontrivial interlaminar stresses in the twisting load case, the assumed base functions $g_i(\eta)$ and $h_i(\eta)$ are chosen independently.

The in-plane stress functions are determined from the initially assumed basis set of out-of-plane functions which must satisfy traction-free conditions at the top and bottom surfaces. The initial out-of-plane functions $g_i(\eta)$ are assumed to be the eigenmodes of a clamped–clamped beam vibration.

The governing equations are obtained from the principle of complementary virtual work.

$$\begin{aligned}
0 &= \iiint u_i \, \delta\sigma_{ij,j} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \\
&= \iiint \{(u_i \, \delta\sigma_{ij})_{,j} - u_{i,j} \, \delta\sigma_{ij}\} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \\
&= \iint_s u_i \, \delta\sigma_{ij} n_j \, \mathrm{d}A - \iiint \frac{1}{2}(u_{i,j} + u_{j,i}) \, \delta\sigma_{ij} \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z.
\end{aligned} \tag{6}$$

By using traction-free boundary conditions and neglecting rigid body motions, one obtains

$$\begin{aligned}
&\iint (\Delta u \, \delta\sigma_{xx} + \Delta v \, \delta\sigma_{yx} + \Delta w \, \delta\sigma_{zx}) \, \mathrm{d}y \, \mathrm{d}z \\
&\quad = \iint \varepsilon_{ij} \, \delta\sigma_{ij} \, \mathrm{d}y \, \mathrm{d}z \quad (\Delta u = C - Bz, \ \Delta v = -\Theta z, \ \Delta w = B/2 + \Theta y),
\end{aligned} \tag{7}$$

where B characterizes the bending of the body in the z–x plane and C characterizes the extension of the body along the x-axis. Θ is the relative angle of rotation about the x-axis.

From the initially assumed out-of-plane basis function set, one can get the in-plane stress functions. The first process is given as follows. Substituting Eq. (3) into Eq. (7), the stresses are expressed in terms of f_i and p_i. The Euler differential equations for f_i and p_i can be obtained from Eq. (7). Thus in-plane stress functions are determined from the initially assumed out-of-plane stress functions $g_i(\eta)$.

In the second process, Kantorovich method is reapplied to the original complementary virtual work principle given in Eq. (7). Substituting the in-plane stress functions f_i and p_i, which were obtained in the first process, into Eq. (7), the enhanced out-of-plane stress functions $g_i(\eta)$ are obtained by solving Euler equations derived from Eq. (7).

The third iteration process is similar to the first one and the fourth process is similar to the second process. In the computer program, n-time iterations can be easily performed since the stress function patterns do not change after the second process. The detailed analysis process can be found in the papers provided by Cho and Yoon (1999) and Cho and Kim (2000).

2.2. Evaluation of the interlaminar strength by the averaged stresses

Most interlaminar stresses show singularity or concentration near the free edges as shown in Fig. 2. Thus, the pointwise stresses at the free edges are not meaningful in the strength analysis because this region is not homogeneous in the micro-length scale. Therefore, the average stress criterion (Whitney and Nuismer, 1974) is employed to evaluate the interlaminar strength of the composite laminates. For example, the average value of interlaminar normal stress (σ_{zz}) can be calculated by the following equation,

$$\bar{\sigma}_{zz} = \frac{1}{h_0}\int_0^{h_0} \sigma_{zz}(\xi, \eta)\,\mathrm{d}\xi. \tag{8}$$

In Eq. (8), h_0 is taken as one ply thickness for all the cases. The concept of the average stress $\bar{\sigma}_{zz}$ is shown in Fig. 2.

2.3. Maximum stress criterion and quadratic delamination criterion

In the present layup optimization problem for maximal strength, the maximum stress criterion was adopted for the in-plane strength criterion, and a quadratic delamination criterion (Brewer and Lagace, 1988) was used for the interlaminar strength criterion. They are given as,

$$-X_c < \sigma_{xx} < X_t, \quad -Y_c < \sigma_{yy} < Y_t, \quad |\sigma_{xy}| < S, \tag{9}$$

$$\left(\frac{\bar{\sigma}_{zz}}{Y_t}\right)^2 + \left(\frac{\bar{\sigma}_{zx}}{S}\right)^2 < 1, \tag{10}$$

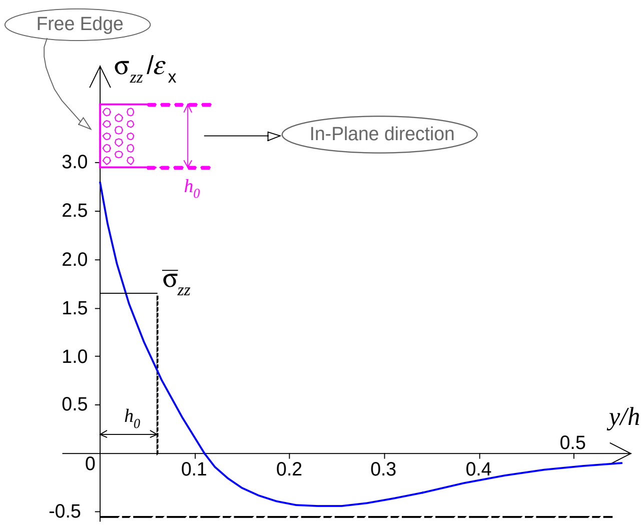

Fig. 2. The concept of averaging stress over the distance (h_0) from free edge.

where, X and Y represent the failure strength along and transverse to the fiber direction, respectively. S denotes the shear strength. Tension and compression are represented by subscripts t and c, respectively. The quantities $\bar{\sigma}_{zz}$ and $\bar{\sigma}_{zx}$ denote the averaged interlaminar stresses near the free edge of the laminates as given in Section 2.2. The quadratic delamination criterion is employed because it is more reliable and compact to apply than the independent maximum average stress criterion. Using the above criteria, we consider the six failure modes: tensile axial, compressive axial, tensile transverse, compressive transverse and interlaminar failure modes.

3. Convex modeling

To consider the uncertainty of the design parameters, the function of the probability distributions should be known. However, the probability function of the scattered distribution requires sufficient data measurements. In the industry, these uncertainty distribution functions may not be available. If the uncertainties under consideration are bounded with respect to the nominal reference values, a convex set from scattering bound of design parameters can be easily constructed. The convex set can be used in the constraint equations of the optimization problem to consider the uncertainties of material data. The procedure for the convex modeling is outlined here to apply to our problems.

Convex modeling needs to be modified to consider properly uncertainty of material properties in evaluating the inerlaminar strength. Thus, the failure surface for interlaminar strength should be approximated more closely to the exact one. Thus in the present study, TPEA is used to build a convex set for the interlaminar strength. TPEA is a methodology for approximating the actual function by using function values and derivatives at the two design points. The sensitivities inserted in the convex modeling are calculated from information from the approximated failure surface.

3.1. Formulation of convex modeling

In this section, formulation of convex modeling is briefly outlined. The detailed explanation can be found in the reference (Kim and Sin, 2000).

Let us assume that $G(D_i)$ is a failure index for optimization. D_i are the uncertain parameters considered in the problem. We consider the uncertainty of material properties in the present study. Then D_1, D_2, D_3 and D_4 are set equal to E_L, E_T, ν_{LT} and G_{LT}, respectively. The failure index $G(D_i)$ can be expanded up to linear terms by considering small parameter changes as follows,

$$G(D_i) = G(D_i^0 + \delta_i) = G(D_i^0) + \sum_{i=1}^{4} \frac{\partial G(D_i^0)}{\partial D_i} \delta_i, \tag{11}$$

where D_i^0 are the nominal values of the uncertain parameters and $|\delta_i| \leqslant \Delta_i$.

The perturbed failure index can be symbolically given as,

$$G(D_i^0 + \delta_i) = G(D_i^0) + \{f\}^{\mathrm{T}}\{\delta\}, \tag{12}$$

where,

$$\{f\}^{\mathrm{T}} = \left[\frac{\partial G(D_i^0)}{\partial D_1}, \frac{\partial G(D_i^0)}{\partial D_2}, \frac{\partial G(D_i^0)}{\partial D_3}, \frac{\partial G(D_i^0)}{\partial D_4}\right], \tag{13}$$

$$\{\delta\}^{\mathrm{T}} = [\delta_1, \delta_2, \delta_3, \delta_4]. \tag{14}$$

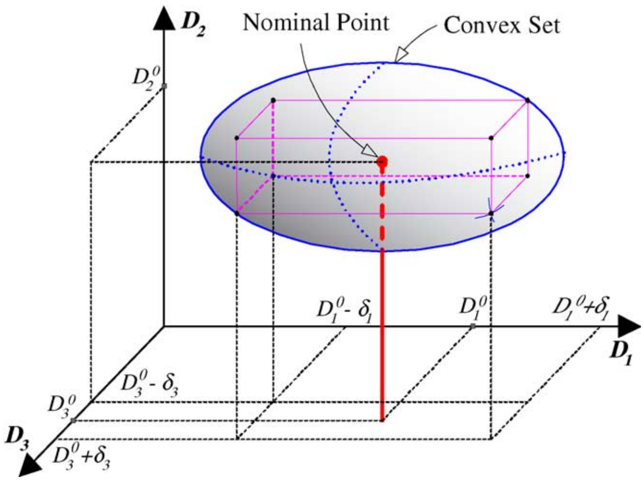

Fig. 3. Schematic of convex set in 3-D configurational space.

If it is assumed that δ_i construct a convex set, then from the linearity of Eq. (12), maximum values are on the boundary of the convex set. The constructed convex set of ellipsoid shape is derived as follows,

$$Z(d) = \left\{ \delta : \sum_{i=1}^{4} \frac{\delta_i^2}{d_i^2} \leqslant 1 \right\}. \tag{15}$$

In Fig. 3, the schematic of the convex set in 3-D configurational space with three uncertain parameters D_1, D_2 and D_3 is displayed.

To increase the accuracy of analysis, the volume of ellipsoid $Cd_1d_2d_3d_4$ should be minimized. Moreover, the corners of the box ($\delta_i = \pm\Delta_i$) have to be on the surface of the ellipsoid. Thus, to obtain d_i, the following Lagrangian should be minimized with respect to d_i and λ,

$$L = Cd_1d_2d_3d_4 + \lambda\left(\frac{\Delta_1^2}{d_1^2} + \frac{\Delta_2^2}{d_2^2} + \frac{\Delta_3^2}{d_3^2} + \frac{\Delta_4^2}{d_4^2} - 1\right). \tag{16}$$

Through the minimization process, d_i can be obtained as,

$$d_i = 2\Delta_i. \tag{17}$$

The problem of maximizing the failure index with material data scattering δ_i can be constructed in the following form,

$$G_{\max} = \mathrm{Max}_{\{\delta\}\in Z(d)}[G(D_i^0) + \{f\}^{\mathrm{T}}\{\delta\}]. \tag{18}$$

The problem can be expressed by the following modified Lagrangian,

$$L(\delta) = \{f\}^{\mathrm{T}}\{\delta\} + \lambda(\{\delta\}^{\mathrm{T}}\{\varepsilon\}\{\delta\} - 1), \tag{19}$$

where $\{\varepsilon\}$ is a diagonal matrix whose diagonal elements are $\varepsilon_{ii} = 1/d_i^2$.

After obtaining the Lagrange multiplier, $\{\delta\}$ for the maximum failure index is obtained as,

$$\{\delta\} = \pm\frac{1}{\sqrt{\{f\}^{\mathrm{T}}\{\varepsilon\}^{-1}\{f\}}}\{\varepsilon\}^{-1}\{f\}. \tag{20}$$

The maximum failure index considering bounded scattered data can be finally obtained as,

$$G_{\max} = G(D_i^0) \pm \sqrt{\{f\}^{\mathrm{T}}\{\varepsilon\}^{-1}\{f\}} = G(D_i^0) \pm \sqrt{\sum_{i=1}^{4}\left[d_i \frac{\partial G(D_i^0)}{\partial D_i}\right]^2}. \tag{21}$$

To construct a convex set for considering the bounded uncertainty of scattered parameters, the sensitivities are computed with a finite difference scheme. That is,

$$\frac{\partial G}{\partial D_i} = \frac{G^{\varepsilon} - G^0}{\varepsilon D_i}\begin{bmatrix}\varepsilon: \text{ small number } (0.0001) \\ D_i: \text{ nominal values of properties}\end{bmatrix}, \tag{22}$$

where, G^0 and G^{ε} are the objective functions at the nominal values of material properties and at the scattered values, respectively.

When the convex model is applied even for the same layup, the failure mode may be altered as the amount of deviations of material properties is changed. Thus, six independent convex sets for one layup are constructed to consider six independent failure modes. The minimum among the failure indices calculated from the six convex sets is assigned as the fitness of the layup.

3.2. Modification of convex modeling for interlaminar strength: TPEA

A TPEA (Fadel et al., 1990) was developed originally for reducing computational cost. Here, it is employed to estimate derivatives in the convex model more accurately. This approximation is a linear approximation with the exponent transform through intermediate variables. The physical variables are transformed to the intervening variables using the relation,

$$Y_i = D_i^{p_i} \quad (i = 1, 2, \ldots, n), \tag{23}$$

where n is the number of design variables and the exponents p_i are determined by matching derivatives of function at the nominal values of material properties.

$$\left.\frac{\partial \tilde{G}}{\partial Y_i}\right|_{\vec{Y}^0} = \left.\frac{\partial G}{\partial Y_i}\right|_{\vec{Y}^0} \Rightarrow \left.\frac{\partial \tilde{G}}{\partial D_i}\right|_{\vec{D}^0} = \left(\frac{D_i^0}{D_i^d}\right)^{p_i - 1} \left.\frac{\partial G}{\partial D_i}\right|_{\vec{D}^d}, \tag{24}$$

where $\vec{D}_i^0$ is the point which consists of nominal values and $\vec{D}_i^d$ is the perturbed point which is at a certain distance from the nominal point.

Finally, the approximated function is obtained by expanding the function at the deviated point as,

$$\tilde{G}(\vec{D}) = G(\vec{D}^d) + \sum_{i=1}^{4}\left\{\frac{(D_i^d)^{p_i - 1}}{p_i} \cdot \left.\frac{\partial G}{\partial D_i}\right|_{\vec{D}^d} \cdot [(D_i)^{p_i} - (D_i^d)^{p_i}]\right\}. \tag{25}$$

Sensitivities required to construct convex model are calculated from the exact function value at the nominal point and approximated function value at the deviated point.

$$\left.\frac{\partial \tilde{G}}{\partial D_i}\right|^{\mathrm{Subst}} = \frac{G(\vec{D}^0) - \tilde{G}(D_1^0, D_2^0, \ldots, D_i^0 - \Delta_i, \ldots, D_n^0)}{\Delta_i}, \tag{26}$$

where the superscript 'Subst' indicates the substituted sensitivity. For 1-D example, the substituted sensitivity corresponds to the slope shown in Fig. 4. The sensitivity value in the modified convex model is now changed by using TPEA given in Eqs. (25) and (26).

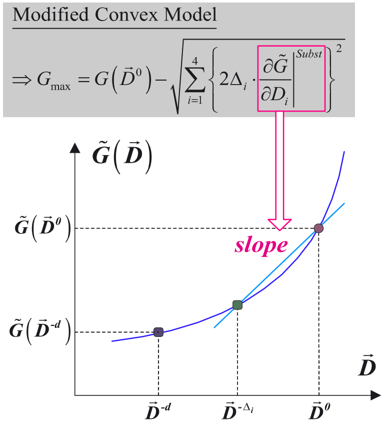

Fig. 4. Sensitivity obtained by using TPEA in 1-D.

4. Genetic algorithm

The design objective of the present study is to obtain layup configurations of symmetric laminates that sustain the maximum applied load under the above-mentioned independent maximum stress criterion. The 16-ply symmetric layup configurations are considered in the present study. Each ply thickness is fixed, and ply orientation angles are limited to 0°, ±30°, ±45°, ±60° and 90°. This limitation makes the layup design a combinatorial optimization problem. Genetic algorithms have been used extensively to solve this type of combinatorial problems. Genetic algorithms are well suited for the problem of layup optimization, and because of the random nature of GA, they easily produce alternative optima in repeated runs. This property is particularly important in layup optimization because widely different layups can have very similar performance (Shin et al., 1989).

Three constraints are applied to the present optimization problem. The first one is the symmetric layup constraint, but this is satisfied automatically by the coding rule in which only half of the laminates are represented in a chromosome. The second constraint is a requirement of the balanced laminate construction, which is intended to reduce or eliminate undesirable extensional–shear coupling and bending–twisting coupling. The third constraint is a limit of four contiguous plies with the same fiber orientation, which reduces the problem of matrix cracking. These constraints are referred to as 'balance constraint' and 'four-contiguity constraint' respectively in the following descriptions. It is not easy to enforce these two constraints in genetic optimization. Penalty function may be used to handle these constraints. But in the present study, a recessive-gene-like repair strategy (Todoroki and Haftka, 1998) is applied with modifications. The key concept of the strategy is to repair the laminate without changing the chromosome.

For the problem with multiple global optima, the optimization process that finds as many optima as possible is required. To find these optima, the multiple elitism strategy (Soremekun et al., 2001), which copies the best designs in current generation into the next generation, is adopted.

4.1. Outline of GA scheme

The flowchart of GA is illustrated in Fig. 5. To represent the ply angles in a layup as genes (in a chromosome), five numbers are introduced with each gene having one of the values of 0, 1, 2, 3 or 4. The gene-0 and gene-4 correspond to 0° and 90° plies, respectively. The occurrences of the first (outermost), third, fifth, *and so on* of gene-1 correspond to +30° while even-number occurrences correspond to −30°. In a

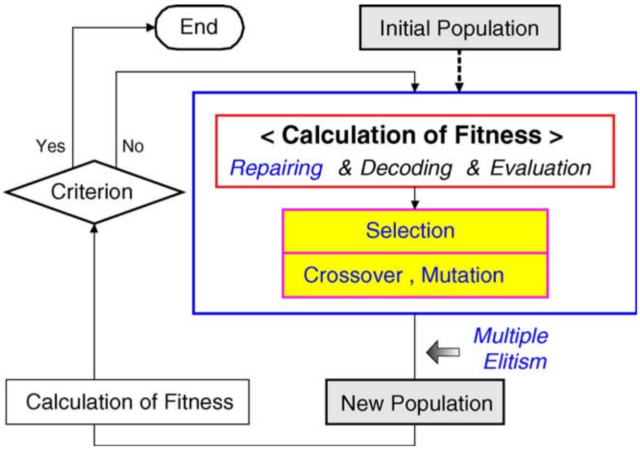

Fig. 5. Flowchart of GA.

similar way, gene-2 and gene-3 represent ±45° and ±60°, respectively. Herein, only half of the plies is represented by the chromosome due to the symmetry constraint of laminates.

The initial population of chromosomes is generated at random. Each chromosome (laminate) is repaired by the following repair strategies and evaluated by the maximum applied load, calculated by using the strength analysis described in the previous section.

The N_e best chromosomes of each generation are always copied into the next generation by multiple elitism. Selection is executed by a linear search through a roulette wheel with slots weighted in proportion to string fitness values.

After selection, a two-point crossover, which is different from a simple crossover, is conducted with a probability value of P_{cr}. Two random cut-points are chosen first, and the offsprings are generated by combining the middle segment of one parent with the outer segment of the other parent. When crossover is not conducted, the selected two parents are copied into the next generation.

Mutation is applied to the chromosomes, except for the elites of the previous generation. The probability of mutation, P_{mu}, is defined as the percentage of the total number of genes in the population. This operator prevents premature loss of important genetic information by randomly altering a chromosome.

4.2. Repair strategy for balance constraint

When the number of gene-1 is odd, the decoded laminate will be unbalanced, or only one unbalanced +30° ply will be in excess in the laminate. The situations will be same for gene-2 and gene-3. In the present study, the strategy for balance constraint is classified into three cases. The repairing procedures are adopted such that they make the least changes in the mechanical behavior of the repaired layup compared to that of the unrepaired layup.

When only one kind of gene among three genes (gene-1, gene-2 and gene-3) violates balance constraint, the innermost gene-1 is changed into gene-0, and the gene-2 is changed into gene-0 or gene-4 with the same probability, and the gene-3 into gene-4, respectively. If there is only one violating gene, the innermost gene-0 or gene-4 is altered to balance the violating gene. When two kinds of genes violate this constraint, the innermost gene among the violating genes is converted into the other kind of gene.

Let us consider a case in which there are three gene species that violate the balance constraint. If the innermost gene among the group of gene-1's and gene-3's is gene-1, the innermost violating gene-1 should be changed to gene-0. Next, the remaining violating genes are gene-2 and gene-3. The next repairing process is to convert the innermost violating gene (among the group of gene-2's and gene-3's) into the remaining angle-ply gene (gene-2 $\leftrightarrow$ gene-3). In the other case in which gene-3 is the innermost instead of gene-1, the innermost gene-3 is converted into gene-4. Consecutively, the innermost violating gene (among the group of gene-1's and gene'2) is switched into the remaining angle-ply gene (gene-1 $\leftrightarrow$ gene-2).

This strategy is similar to that of Todoroki's version but not the same. For example, in the present repairing strategy, the repairing process for balance constraint precedes the four-contiguity constraint repairing process. Thus, extra effort is not required to reconsider the four-contiguity constraint in checking the balance constraint. In addition, three kinds of angle-ply genes are considered in the present study (±30°, ±45°, ±60°), whereas only one kind of angle-ply genes is considered in Todoroki's study (±45°). Thus, the consideration of balance constraint is more complicated in the present strategy than in Todoroki's.

If the repair procedure does not change the laminate but changes the genes only, it can prevent beneficial changes which occur as a result of two or more consecutive mutations. For example, consider a case when it is beneficial to transform the chromosome [13130202] corresponding to the $[30/60/-30/-60/0/45/0/-45]_s$ laminate into [13130404] corresponding to the $[30/60/-30/-60/0/90/0/90]_s$ laminate. When the chromosome [13130202] mutates into the chromosome [13130204], the repair system reverses the change, and the chromosome still corresponds to the $[30/60/-30/-60/0/45/0/-45]_s$ laminate. In this case, the repair procedure does not change the chromosome. One additional mutation in a future generation can transform the gene to [13130404], and the innermost gene-4 will now be developed into the 90° ply. The innermost gene-4 acts like a recessive gene.

4.3. Repair strategy for four-contiguity constraint

This constraint is concerned with only gene-0 and gene-4. When there are more than four contiguous genes, the innermost one of the contiguous genes is converted into other suite of gene. For example, [02444420] is repaired into [2444020].

For the innermost genes, this repair procedure should be modified. When the two innermost genes are of the same kind, there are already four contiguous plies with the same orientation in the middle of the laminate due to the layup symmetry. Therefore, the repair process is not allowed to stack more than two genes in the innermost position within a laminate. When the innermost genes violate this rule, the gene value of the innermost ply is converted into another kind of gene. For example, [04104100] is converted into [04104104].

4.4. Multiple elitism

The schematic of multiple elitism is shown in Fig. 6. The top designs (elites) from the parent population are selected and placed into the new population. The child designs required to fill the remainder of the new population are created from the remaining parents that have not been selected as multiple elites, and then placed into the new population. This selection scheme is computationally less intensive because fewer child designs require fitness computation. The number of elites to be copied into the next generation (N_e) is determined by the following equation, in which the more elites are selected as the population size increases,

$$N_e = \left\lfloor \frac{\text{PopSize} + 5}{4} \right\rfloor, \tag{27}$$

where, PopSize is the population size and the symbol "$\lfloor\ \rfloor$" (floor) indicates the largest integer smaller than or same as the number in the symbol. For example, when population size is 25, seven elites are selected and copied into the next generation.

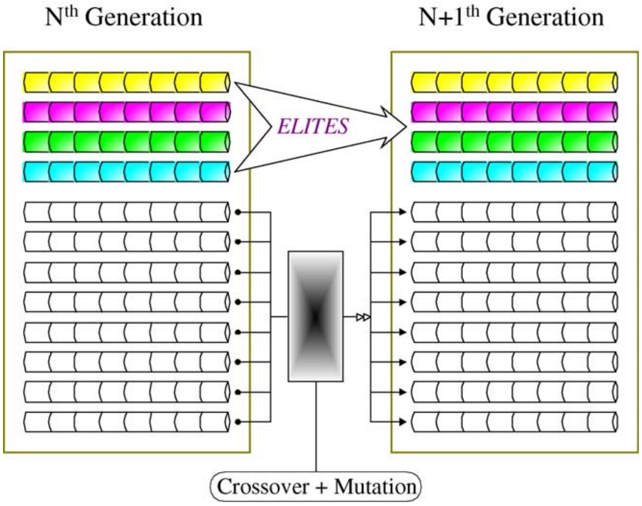

Fig. 6. Schematic of multiple elitism.

4.5. Criteria for evaluating the performance of GA

If the optimized results have been obtained, the performance of GA can be estimated for various parameters. In the following study, four different criteria, which was previously proposed by Soremekun et al., are applied to assess the performance of the present GA in the layup optimization problem. The first criterion is the normalized cost per genetic search, C_n, determined by

$$C_n = \frac{N_g N_c}{R} \left(R = \frac{N_{op}}{N_r} \right), \tag{28}$$

where N_g is the number of generations per run, N_c is the number of child designs created in each generation, and R is reliability. If GA is run N_r times and succeeds in finding at least one of the several global optima N_{op} times of these runs, then the reliability R is calculated as the equation in the parenthesis of Eq. (28).

The second criterion is the average number of optima found per genetic search:

$$A_{N_0} = \frac{\sum_{i=1}^{N_r} N_0^i}{N_r}, \tag{29}$$

where N_0^i is the number of optima found in the ith optimization run. In the present study, there are three global optima for the case of extension and there are three pseudo-optima for the cases of bending. Thus, we choose three as the maximum number of this criterion for all cases.

The third criterion is defined as the cost per optimum found:

$$C_0 = \frac{N_g N_c}{A_{N_0}}. \tag{30}$$

The final criterion is the final population richness, which helps to monitor how the GA exploits global optimum regions of the design space. Final population richness P_r is defined as,

$$P_r = \frac{N_{\Delta f}}{\text{PopSize} \cdot N_r}, \tag{31}$$

where $N_{\Delta f}$ is the number of members in the final population of each run with fitness values within a certain small amount (Δf) of the optimum.

5. Results

In the present study, graphite–epoxy composite laminates under extension, bending, and twisting are considered. We consider symmetric 16-ply layup configurations. The material properties and strength data are as follows,

$$\begin{aligned}
&E_1 = 207\ \text{GPa}, \quad E_2 = E_3 = 5\ \text{GPa}, \quad G_{12} = G_{13} = 2.6\ \text{GPa}, \\
&G_{23} = 1.8\ \text{GPa}, \quad \nu_{12} = \nu_{13} = 0.25, \quad \nu_{23} = 0.45, \\
&X_\text{t} = 1035\ \text{MPa}, \quad X_\text{c} = 689\ \text{MPa}, \\
&Y_\text{t} = 41\ \text{MPa}, \quad Y_\text{c} = 117\ \text{MPa}, \quad S = 69\ \text{MPa},
\end{aligned}$$

where subscripts 1, 2, and 3 indicate the x-, y- and z-direction in the space, respectively.

In the calculation of the interlaminar stresses, two term expansion ($n = 2$) in Eqs. (4) and (5), is used and three iterations are performed in the extended Kantorovich method. These selections are sufficient to calculate stresses reliably.

Various parameters—population size, probability of mutation, and probability of crossover, and so on—are given in Table 1. As previously stated, four criteria in Section 4.5 are adopted to evaluate the numerical performance of GA. In the evaluation of $N_{\Delta f}$ in Eq. (31), we consider the designs whose fitnesses are within the deviation of 9.2% from the optimal one for extension and 6.4% for bending and 12.4% for twisting.

5.1. For the case of extension

In this case, the three global optima are obtained as $[0/0/0/30/0/0/0/-30]_\text{s}$, $[0/0/0/30/0/0/-30/0]_\text{s}$ and $[0/0/30/0/0/0/-30/0]_\text{s}$, regardless of whether the uncertainty in material properties is considered. The optimal fitness is 1.383×10^6 for nominal properties. When the uncertainties of material properties are considered, the fitness is 1.375×10^6, which is 0.58% smaller than the optimal fitness for nominal material values. The deviation of all the material properties is set to 5% from nominal values. The failure mode of the optimal is the tensile axial direction (fiber breakage) for both cases.

The results of parametric evaluation in the application of GA are shown in Fig. 7, the plot only for nominal material properties. For a population size greater than 15, as the population size increases, GA requires more cost but finds more optima. By the implementation of multiple elitism, the average number of optima found converges to the maximum value 3.

The sensitivities for optimal laminates are given in Table 2. The strength sensitivity for each material property is given in the descending order. Modulus E_L has the largest sensitivity, and E_T is the second, and this result depends on the loading condition. In the extension problem, changes of ν_LT and G_LT do not affect the strength of laminated composites significantly.

Table 1
Parameters used in the application of GA

Parameters	Values
Chromosome length	8
Upper limit of generation	100
Number of runs (N_r)	30
Population size	7–50
Probability of mutation	0.1
Probability of crossover	1.0

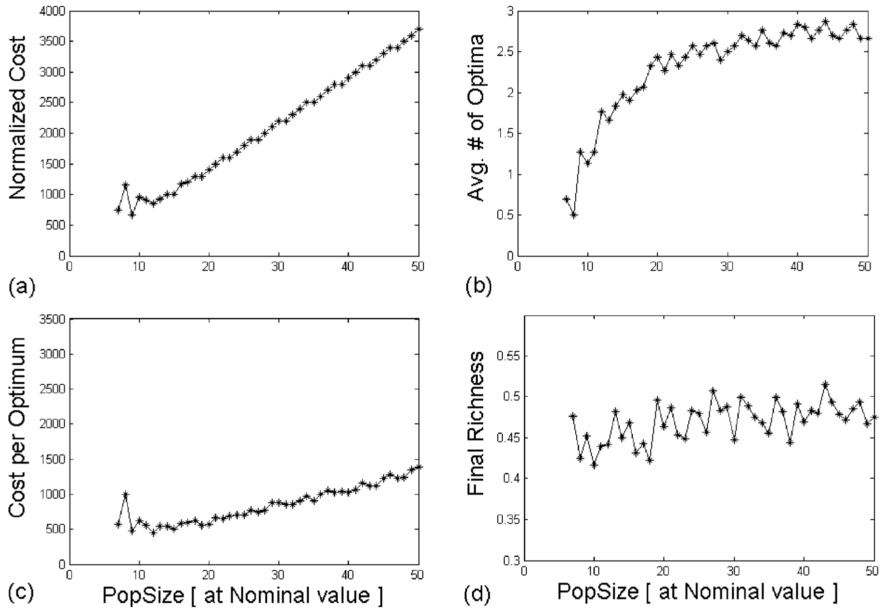

Fig. 7. Parametric study for population size (at the nominal values of material properties, for the case of extension).

Table 2
Sensitivities with respect to the material properties (for the case of extension)

Material properties	Normalized sensitivities
E_L	-5.886×10^4
E_T	$+3.965\times10^4$
ν_{LT}	-0.915×10^4
G_{LT}	$+1.921\times10^4$

5.2. For the case of bending

In this case, the global optimized layup is obtained as $[0/0/0/30/0/0/0/-30]_s$, and the second and third optimum are $[0/0/0/30/0/0/-30/0]_s$ and $[0/0/30/0/0/0/-30/90]_s$. The optimal fitness is 2.658×10^{-2}, and the second and third optimal fitness are 2.657×10^{-2} and 2.653×10^{-2}, respectively. The discrepancies of the second and third optimal fitness from the first one are 0.054% and 0.146%, respectively. Because the discrepancies are very small, the second and third optima can also be counted as one of the multiple pseudo-optima. The failure mode of all three optima is the axial direction compression. Considering the uncertainties of the material properties, optimal fitness decreases 0.23% from optimal one at nominal properties. The second and third layup are not changed.

The results of a parametric study for evaluating GA's performance are shown in Fig. 8. The performance for this case is similar to that of the case of extension except for the average number of optima found. In bending, the pattern of the third optimum layup is different from that of the first two optima. Thus, GA has some difficulty in finding all three optima in every run, even though the population size increases. But considering the small number for the generation limit specified, GA in the present study shows excellent performance.

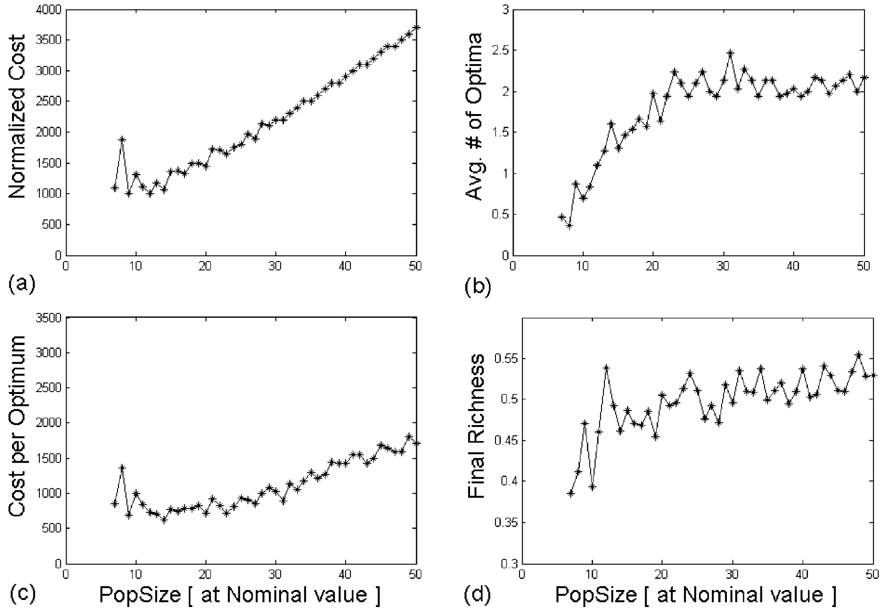

Fig. 8. Parametric study for population size (at the nominal values of material properties, for the case of bending).

Table 3
Sensitivities with respect to the material properties (for the case of bending)

Material properties	Normalized sensitivities
E_L	-4.713×10^{-4}
E_T	$+0.703\times10^{-4}$
ν_{LT}	-0.470×10^{-4}
G_{LT}	$+4.010\times10^{-4}$

The sensitivities for optimal laminates are given in Table 3. In this problem, deviations of E_L and G_{LT} from the nominal values have significant effects on the strength of laminated composites.

5.3. For the case of twisting

At the nominal properties, the global optimized layup is obtained as $[30/-30/45/0/60/90/-60/-45]_s$, the second and third optimum are $[90/60/-60/30/45/-30/0/-45]_s$ and $[0/60/-60/30/45/-30/90/-45]_s$, respectively. The optimal fitness is 10.84×10^{-4} and the second and third optimal fitness are 7.645×10^{-4} and 6.688×10^{-4}, respectively. For checking the efficiency of multiple elitism, the three optima are treated as the multiple optima. The failure mode of all three optima is the interlaminar failure mode. Considering the uncertainties of the material properties, the global optimum layup is changed. In detail, the layup $[60/45/-60/-45/90/90/90/0]_s$ takes the first rank with its fitness value, 5.510×10^{-4}. The failure mode is not changed. The second and third layup are $[60/45/-60/-45/90/90/45/-45]_s$ and $[60/45/-60/-45/90/90/0/90]_s$, respectively.

In Fig. 9, GA's performance accounting for the uncertainties of material properties is shown. For this case, the second optimum layup was difficult to find for the same reason as that for the case of bending. The decreased fitness, the term in the root of Eq. (21), of the optimum at the nominal material properties is much larger than that of the optimum with uncertainties. Thus the old optimum is replaced by the new

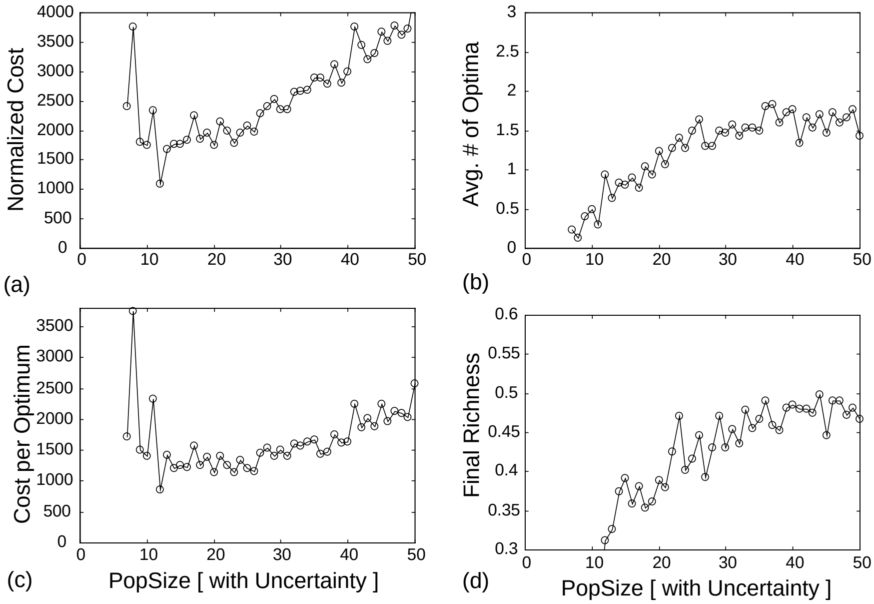

Fig. 9. Parametric study for population size (with the uncertainty of material properties, for the case of twisting).

Table 4
Sensitivities with respect to the material properties (for the case of twisting)

Material properties	Normalized sensitivities	
	Old optimum	New optimum
E_L	$+6.465\times10^{-4}$	$+3.341\times10^{-4}$
E_T	-2.576×10^{-4}	-4.831×10^{-4}
ν_{LT}	$+0.077\times10^{-4}$	$+0.185\times10^{-4}$
G_{LT}	-5.959×10^{-4}	-10.099×10^{-4}

optimum. The perturbed amount is calculated by the sensitivities in Table 4. To promote readers' comprehension, the averaged interlaminar normal and shear stresses in the old optimum layup are plotted in Figs. 10 and 11, while those in the new optimum layup are plotted in Figs. 12 and 13. The dash-dot lines represent the interlaminar stresses when the most sensitive material property has a 5% deviation.

6. Conclusion

Extended Kantorovich method could provide efficient and accurate interlaminar stresses near the free edges. With these stresses, the layup of composite laminates could be optimized for maximum strength, considering the interlaminar strength. The layup optimization, which may not be treated by traditional gradient-based optimization techniques, could be executed with the help of a genetic algorithm. It was demonstrated that GA with repair strategy works well in handling constraints in the layup optimizations of composite laminates. GA with multiple elitism was able to find more solutions near the global optimum. This result is important because the designer can have more flexibility in selecting the layup of composite laminates.

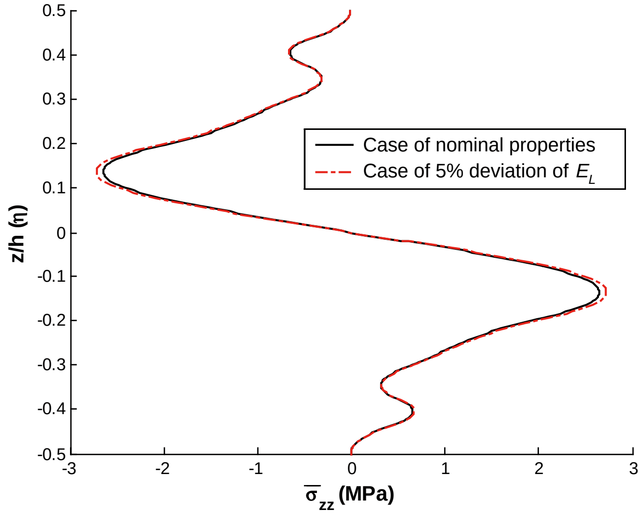

Fig. 10. Averaged interlaminar normal stress in [30/−30/45/0/60/90/−60/−45]$_s$ layup under twisting load.

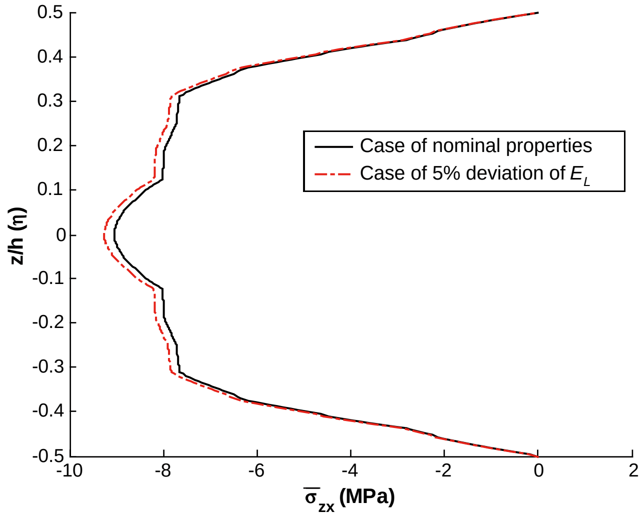

Fig. 11. Averaged interlaminar shear stress in [30/−30/45/0/60/90/−60/−45]$_s$ layup under twisting load.

The bounded uncertainty could be easily considered in the optimization procedure by imposing the scattering bounds in the input process. In the present problem, the optimal layup configuration is not changed in the cases of the extension and bending loads even though the uncertainties of material properties are considered. However, it should be emphasized that in the case of twisting, the optimal layup was changed when the uncertain material properties was considered. Thus, to obtain a reliable strength design, it is recommended that the new optimal layup configuration considering uncertain material properties replace the layup configuration for nominal material properties under twisting loads.

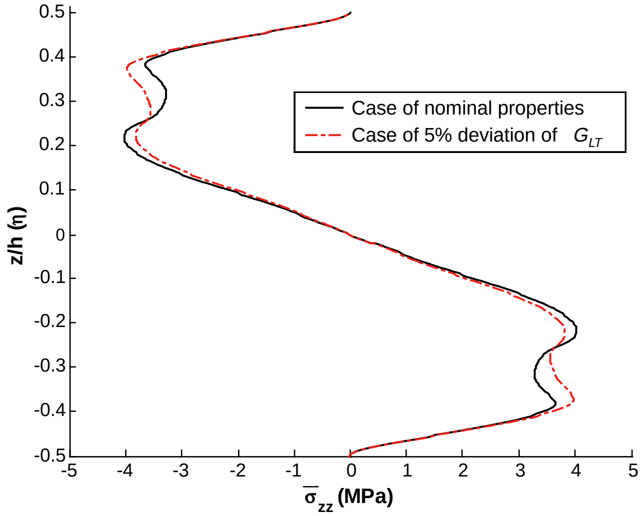

Fig. 12. Averaged interlaminar normal stress in $[60/45/-60/-45/90/90/90/0]_s$ layup under twisting load.

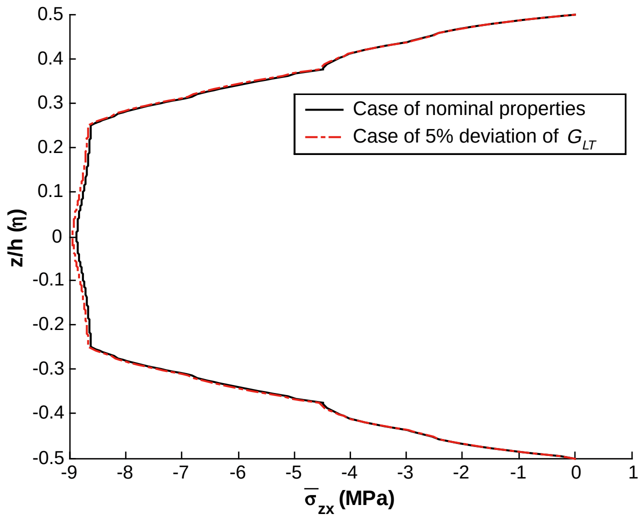

Fig. 13. Averaged interlaminar shear stress in $[60/45/-60/-45/90/90/90/0]_s$ layup under twisting load.

The methodology proposed in the present study can be used as a powerful tool in the layup design of composite laminates.

Acknowledgement

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